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A VELOCITY AMBIGUITY WHICH DEPENDS ON TARGET ROTATION RATE,(U)
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A VELOCITY AMBIGUITY WHICH DEPENDS ON TARGET ROTATION RATE

R. C. RAUP
Group 91

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ABSTRACT

A constant force, zero torque deterministic model for CW radar observations of a rotating rigid target is presented. A velocity ambiguity which depends on the target's rotation rate develops when the target's scattering configuration is unknown and arbitrary.

I. INTRODUCTION

To allow meaningful target trajectory estimation problems in noisy environments, it is desirable to generalize the low dimensional constant range or constant velocity models designed for rotationally stable targets ([7], sect. 9.1). These low dimensional models have few parameters, but they may provide a poor approximation of the radar observations over the longer time intervals used to accumulate useable signal energy in deep space satellite surveillance problems.

The effect of target rotation on line of sight velocity estimates can be analyzed for specific scatterer configurations (e.g., [1], [2]). More generally, a probabilistic target model can be developed incorporating a doppler scattering function ([7], sect. 11.1). By design, a statistic of the model's doppler scattering function, for example its mean, determines the target velocity [3].

In the absence of such a priori knowledge concerning scatterer interactions, a velocity ambiguity develops in the generalized target model which depends on the transmitter frequency and the target rotation rate. This ambiguity is not due to the periodic envelopes used in some transmitter waveforms or the uniform sampling of continuous time functions which are two common sources of ambiguity.

We begin by recalling the constant force, zero torque model for observations of a rigid body. We suppose that the observations are made by a coherent CW radar. The velocity ambiguity is then developed as a function of radar wavelength and target rotation period.

II. THE OBSERVATION MODEL

Suppose (using complex notation throughout) that a CW radar transmits a signal

$$m(t) = \exp j(-\omega t) \tag{1}$$

where $\exp j(x)$ denotes e^{jx}. The response due to a single scatterer k may be written in terms of $\tau_k(t)$, the roundtrip propagation time for the waveform as

$$S_k(t) = A_k(t) \exp j\,(-\omega(t - \tau_k(t))) \tag{2}$$

and (by superpositioning in a linear radar) as

$$S(t) = \sum_{k=1}^{N} S_k(t) = \sum_{k=1}^{N} A_k(t) \exp j\,(-\omega(t - \tau_k(t))) \tag{3}$$

for N scatterers.

Let $w_k(t)$ be the signed magnitude of the projection of the vector from mass center to scatterer k onto the line of sight. We suppose that $w_k(t)$ is signed so that $w_k(t)>o$ when the range to scatterer k is greater than the range to the mass center. Then the propagation delay $\tau_k(t)$ for distant targets may be written approximately in terms of the range r(t) to the mass center as

$$\tau_k(t) = \frac{2}{c}\,(r(t) + w_k(t)) \tag{4}$$

where c is the waveform propagation velocity. If the range function r(t) has a convergent power series expansion then

$$r(t) = \sum_{n=0}^{\infty} \frac{r^{(n)}(0)\, t^n}{n!} \tag{5}$$

The coefficient $r^{(1)}(0)$ has the physical significance of the line of sight velocity at t=0. To expose this we write r(t) in the form

$$r(t) = v_0\, t + z(t) \tag{6}$$

where

$$z(t) = \sum_{n \neq 1} \frac{r^{(n)}(0)t^n}{n!}\text{, and } v_0 = r^{(1)}(0) \tag{7}$$

The function associated with the demodulated target observation on the time interval $[-\bar{t}, \bar{t}]$ can be written as

$$u(t) = S(t) \exp j\,(\omega t)$$

$$= \exp j\left(\frac{2\omega}{c}(v_0 t + z(t))\right)\left\{\sum_{k=1}^{N} A_k(t) \exp j\left(\frac{2\omega}{c} w_k(t)\right)\right\} \tag{8}$$

for $|t|<\bar{t}$. If we assume that the forces acting on the satellite produce negligible torque over $\left[-\bar{t}, \bar{t}\right]$, then the factor in brackets will have periodic magnitude and phase. Thus for some $p(t)$, $A(t)$ and T with $p(t) = p(t+T)$ and $A(t) = A(t+T)$,

$$u(t) = A(t) \exp j\left(\frac{2\omega}{c}(v_0 t + z(t) + p(t))\right) \tag{9}$$

Expression (9) is valid in virtually all cases where satellite torques are small and antenna pointing angles are approximately constant on the time interval $[-\bar{t}, \bar{t}]$.

The function $p(t)$ is generally sophisticated because it is determined by the interaction of a complex configuration of scatterers. In this note we do not assume a priori knowledge of the scatterer interactions. The period T can often be estimated reliably and so, from the spectral viewpoint, the assumptions imply that the fundamental frequency of $p(t)$ is available, but the distribution of its energy among harmonics is unknown. For a large class of practical problems stronger assumptions cannot be made.

III. THE AMBIGUITY

It is now shown that there are a countable number of "targets" rotating with period T and traveling at different line of sight velocities which produce the same observation $u(t)$. Each "target" corresponds to a different choice for the distribution of energy among the harmonics of the spectral representation of $p(t)$. Thus from the observed valued of $u(t)$ the velocity v_0 cannot be determined unambiguously. Define

$$\bar{v}(t) = \frac{2\pi k}{T} t \tag{10}$$

for integers k. Then

$$\text{(i)} \quad \operatorname{expj}(\bar{v}(t)) = \operatorname{expj}(\operatorname{mod}(\bar{v}(t), 2\pi)), \text{ and}$$

$$\text{(ii)} \quad \operatorname{mod}(\bar{v}(t), 2\pi) \text{ is periodic with period } \frac{T}{k} \tag{11}$$

where mod (x,y) denotes x modulo y. The first property is a consequence of the polar notation and the second property is a consequence of the definition of $\bar{v}(t)$.

Multiply (9) by one in the form

$$\operatorname{expj}(-\bar{v}(t)) \cdot \operatorname{expj}(\operatorname{mod}(\bar{v}(t), 2\pi)) \tag{12}$$

to obtain

$$u(t) = A(t) \operatorname{expj}\left(\frac{2\omega}{c} z(t) + \left(\frac{2\omega}{c} v_0 - \frac{2\pi k}{T}\right) t + \left(\frac{2\omega}{c} p(t) + \operatorname{mod}(\bar{v}(t), 2\pi)\right)\right) \tag{13}$$

We set $$v_k = \left(\frac{2\omega}{c} v_0 - \frac{2\pi k}{T}\right) \frac{c}{2\omega} \tag{14}$$

and $$\overline{p}_k(t) = \left(\frac{2\omega}{c} p(t) + \text{mod}\,(\overline{v}(t), 2\pi)\right) \frac{c}{2\omega} \tag{15}$$

so that $$u(t) = A(t)\,\text{expj}\left(\frac{2\omega}{c}\left(v_k t + z(t) + \overline{p}_k(t)\right)\right) \tag{16}$$

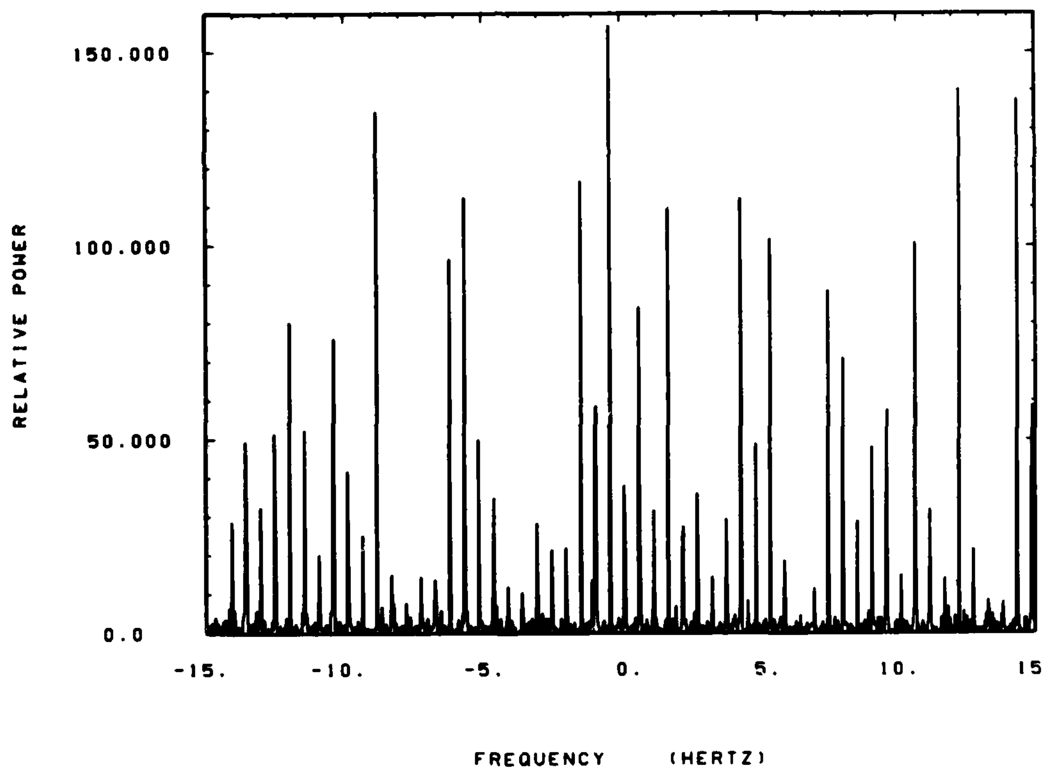
From observations of $u(t)$, the most that any parameter estimation scheme will yield under our assumptions is, from (16), the countable class of velocities v_k and phase functions $\overline{p}_k(t)$ producing the observations. Thus v_0 is only determined ambiguously through (14). We can, however, determine

$$v = \text{mod}\,\left(v_0, \frac{c\pi}{\omega T}\right) \tag{17}$$

The ambiguity $\frac{c\pi}{\omega T}$ is too small for many practical satellite surveillance problems. The ambiguity for a 1295 MHz radar observing a satellite rotating with a period of 2 seconds is less than 6 cm/sec.

IV. DISCUSSION

The figure is a periodogram (magnitude squared of a discrete Fourier transform) of a sampled observation of a rotating target. The observation was made with the 1295 MHz Millstone Hill radar. The nearly discrete spectrum corresponds to an observation $u(t)$ modeled by (16) with $z(t)$ driven to zero by a post processing tracker [4]. The velocity ambiguity problem discussed in this note is related to determining which of the discrete spectral lines corresponds to the doppler shifted transmitter waveform.



The implications of equation (16) are clearly independent of the scheme used to estimate the model parameters. For example, suppose that $z(t)=0$ and that $p(t)$ is bandlimited. Then the likelihood function for estimation of the parameters could be constructed. The likelihood function will not be unimodal but will have local maxima of equal amplitude separated by $\frac{c\pi}{\omega T}$ units along the velocity dimension.

There are many ways of strengthening the assumptions on $p(t)$ to circumvent the ambiguity. For example, $p(t)$ and the phase of (16) may be of known smoothness and sufficiently sampled in a low noise environment. Then the phase of (16) can be recovered (unwrapped) from its value modulo 2π [6]. Assuming that $z(t)$ is polynomial, only one of the $\overline{p}_k(t)$ will be smooth and thus $p(t)$ and v_0 will be determined. Alternately, the transmitter may have some bandwidth, altering assumption (1). The use of transmitter bandwidth is preferred when noise is a problem in order to maintain the generality of the model (9).

In practice the countable possibilities for v_0 are reducible to a small number, but rarely can v_0 be determined uniquely without transmitter bandwidth. This is because external bounds on v_0 are usually larger than the ambiguity. Typical bounds on v_0 come from the observed short term change in absolute target range determined by pulse transit time, or perhaps from trajectory constraints.

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